

EXAMEN d'optimisation

(Durée 2h)

Documents autorisés, calculatrice autorisée. Nombre de page : 1.

Exercice n°1 (4 points)

1. Qu'est-ce qu'un point d'inflexion et comment le déterminer?
2. Qu'est-ce qu'une matrice définie positive ?
3. Les fonctions suivantes sont-elles convexes sur le domaine D
 - a. $F(x) = \ln(x)$ $D =]0, +\infty[$
 - b. $F(x) = x^2 + y^2 - 2x - 4y + 6$ $D =]-\infty, +\infty[$

Exercice n°2 (3 points)

$$f(x, y, z) = x^2 + (x - y)^2 + 3(y - z)^2 - 2x + 6y - 6z$$

Rechercher le minimum de cette fonction

Exercice n°3 (3 points)*méthode fonctionnelle à minimiser*

Formulez le problème d'optimisation suivant :

Soit C un cercle donné de centre O et de Rayon R connu. Parmi tous les rectangles inscrits dans ce cercle définir celui qui est d'aire maximale.

Exercice n°4 (6 points)*pb d'optin.*

On construit un container rectangulaire du volume $V = 10m^3$ pour stocker du ciment. Etant donné le coût des matériaux pour fabriquer le fond $C_1 = 50 \text{ euros}/m^2$, le coût des matériaux pour fabriquer les parois de côté $C_2 = 30 \text{ euros}/m^2$ et le coût des matériaux pour fabriquer le couvercle $C_3 = 60 \text{ euros}/m^2$.

Déterminer les dimensions du container telles que le coût total soit minimum. Vous utiliserez la méthode du gradient conjugué.

Exercice n°5 (4 points)Trouver un point minimum de la fonction à $\varepsilon = 10^{-2}$

$$f(x) = (10x^3 + 3x^2 + x + 5)^2$$

On donne le point de départ $x_0 = 2$

Utiliser la méthode d'élimination des intervalles par la méthode de la section d'or.

EXAMEN d'optimisation

(Durée 2h)

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Questions de cours (4 points)

1. Qu'est-ce qu'un point d'inflexion et comment le déterminer? *point de selle entre 2 montées*
2. Qu'est-ce qu'une matrice définie positive? *matrice Hessienne*
3. Soit en un point $x = \bar{x}$ le gradient $\nabla f(\bar{x}) = 0$. Que peut-on dire du point $\bar{x}$ si: *dérivée seconde de la fnc. obj.*
- a) $f(x)$ est une fonction convexe? *$x^T H x > 0$*
- b) $f(x)$ est une fonction concave? *$H < 0$*
- c) la matrice $\nabla^2 f(\bar{x})$ est définie négative? *$= b$*

Exercice n°1 (6 points)

$$f(x) = 4x_1^2 + 3x_2^2 - 4x_1x_2 + x_1$$

- 1- Développer la méthode de la plus forte pente avec γ optimal afin de trouver le minimum de la fonction
- 2- Développer la méthode du gradient conjugué (méthode de Polak et Ribière) avec γ optimal afin de trouver le minimum de la fonction

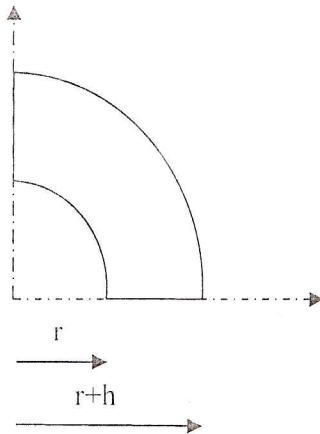
Exercice n°2 (6 points)

Le problème consiste à minimiser $P(r,h)$ d'un domaine annulaire dont l'aire A est fixée, r est le rayon interne et $(h+r)$ le rayon extérieur.

Où $P(r,h) = \pi/2 (2r + h) + 2h$ est le périmètre du domaine annulaire et $A(r,h) = \pi/4 (2r + h) h$ est l'aire.

On impose une contrainte $A(r,h) = L^2$.

- 1- déterminer la valeur de h en fonction de L en éliminant dans un premier temps r dans la relation de l'aire (on utilise la contrainte) et en utilisant la méthode de substitution.
- 2- Refaite cette détermination en utilisant la méthode des multiplicateurs de Lagrange (on dérive la fonctionnelle de Lagrange par les trois variables que sont r , h et Ψ).



$$\begin{aligned} & x_0, x_1 \quad f(x_0) \quad f(x_1) \\ & 0,618 = \frac{x_2 - x_1}{x_0 - x_1} \quad f(x_2) \\ & 0,618 = \frac{x_3 - x_2}{x_1 - x_2} \leftarrow d_1 \end{aligned}$$

Exercice n°3 (4 points)

Trouver un point minimum de la fonction à $\varepsilon = 10^{-2} = 0,01$

$$f(x) = (10x^3 + 3x^2 + x + 5)^2$$

On donne le point de départ $x_0 = 2$

Utiliser la méthode d'élimination des intervalles par la méthode de la section d'or.

$$\begin{aligned} & \text{Initial interval: } [1,99; 2,01] = [x_{\min}, x_{\max}] \\ & \text{Iteration 1: } L_1 = [2; 2,01] \\ & \text{Iteration 2: } L_2 = [2,005; 2,01] \\ & \text{Iteration 3: } L_3 = [2,0025; 2,01] \\ & \text{Iteration 4: } L_4 = [2,00125; 2,01] \\ & \text{Iteration 5: } L_5 = [2,000625; 2,01] \\ & \text{Iteration 6: } L_6 = [2,0003125; 2,01] \\ & \text{Iteration 7: } L_7 = [2,00015625; 2,01] \\ & \text{Iteration 8: } L_8 = [2,000078125; 2,01] \\ & \text{Iteration 9: } L_9 = [2,0000390625; 2,01] \\ & \text{Iteration 10: } L_{10} = [2,00001953125; 2,01] \\ & \text{Iteration 11: } L_{11} = [2,000009765625; 2,01] \\ & \text{Iteration 12: } L_{12} = [2,0000048828125; 2,01] \\ & \text{Iteration 13: } L_{13} = [2,00000244140625; 2,01] \\ & \text{Iteration 14: } L_{14} = [2,000001220703125; 2,01] \\ & \text{Iteration 15: } L_{15} = [2,0000006103515625; 2,01] \\ & \text{Iteration 16: } L_{16} = [2,00000030517578125; 2,01] \\ & \text{Iteration 17: } L_{17} = [2,000000152587890625; 2,01] \\ & \text{Iteration 18: } L_{18} = [2,0000000762939453125; 2,01] \\ & \text{Iteration 19: } L_{19} = [2,00000003814697265625; 2,01] \\ & \text{Iteration 20: } L_{20} = [2,000000019073486328125; 2,01] \\ & \text{Iteration 21: } L_{21} = [2,0000000095367431640625; 2,01] \\ & \text{Iteration 22: } L_{22} = [2,00000000476837158203125; 2,01] \\ & \text{Iteration 23: } L_{23} = [2,000000002384185791015625; 2,01] \\ & \text{Iteration 24: } L_{24} = [2,0000000011920928955078125; 2,01] \\ & \text{Iteration 25: } L_{25} = [2,00000000059604644775390625; 2,01] \\ & \text{Iteration 26: } L_{26} = [2,000000000298023223876953125; 2,01] \\ & \text{Iteration 27: } L_{27} = [2,0000000001490116119384765625; 2,01] \\ & \text{Iteration 28: } L_{28} = [2,00000000007450580596923828125; 2,01] \\ & \text{Iteration 29: } L_{29} = [2,000000000037252902984619140625; 2,01] \\ & \text{Iteration 30: } L_{30} = [2,0000000000186264514923095703125; 2,01] \\ & \text{Iteration 31: } L_{31} = [2,00000000000931322574615478515625; 2,01] \\ & \text{Iteration 32: } L_{32} = [2,000000000004656612873077392578125; 2,01] \\ & \text{Iteration 33: } L_{33} = [2,0000000000023283064365386962890625; 2,01] \\ & \text{Iteration 34: } L_{34} = [2,00000000000116415321826934814453125; 2,01] \\ & \text{Iteration 35: } L_{35} = [2,000000000000582076609134674072265625; 2,01] \\ & \text{Iteration 36: } L_{36} = [2,0000000000002910383045673370361328125; 2,01] \\ & \text{Iteration 37: } L_{37} = [2,00000000000014551915228366851806640625; 2,01] \\ & \text{Iteration 38: } L_{38} = [2,000000000000072759576141834259033203125; 2,01] \\ & \text{Iteration 39: } L_{39} = [2,0000000000000363797880709171295166015625; 2,01] \\ & \text{Iteration 40: } L_{40} = [2,00000000000001818989403545856475830078125; 2,01] \\ & \text{Iteration 41: } L_{41} = [2,000000000000009094947017729282379150390625; 2,01] \\ & \text{Iteration 42: } L_{42} = [2,0000000000000045474735088646411895751953125; 2,01] \\ & \text{Iteration 43: } L_{43} = [2,00000000000000227373675443232059478759765625; 2,01] \\ & \text{Iteration 44: } L_{44} = [2,000000000000001136868377216160297393798828125; 2,01] \\ & \text{Iteration 45: } L_{45} = [2,0000000000000005684341886080801486968994140625; 2,01] \\ & \text{Iteration 46: } L_{46} = [2,00000000000000028421709430404007434844970703125; 2,01] \\ & \text{Iteration 47: } L_{47} = [2,000000000000000142108547152020037174224853515625; 2,01] \\ & \text{Iteration 48: } L_{48} = [2,0000000000000000710542735760100185871124267578125; 2,01] \\ & \text{Iteration 49: } L_{49} = [2,00000000000000003552713678800500929355621337890625; 2,01] \\ & \text{Iteration 50: } L_{50} = [2,000000000000000017763568394002504646778106689453125; 2,01] \\ & \text{Iteration 51: } L_{51} = [2,0000000000000000088817841770012523233890533447265625; 2,01] \\ & \text{Iteration 52: } L_{52} = [2,00000000000000000444089208850062616169452667236328125; 2,01] \\ & \text{Iteration 53: } L_{53} = [2,000000000000000002220446044250313080847263336181640625; 2,01] \\ & \text{Iteration 54: } L_{54} = [2,0000000000000000011102230221251565404236316680908203125; 2,01] \\ & \text{Iteration 55: } L_{55} = [2,00000000000000000055511151106257827021181583404541015625; 2,01] \\ & \text{Iteration 56: } L_{56} = [2,000000000000000000277555755531289135105907917022705078125; 2,01] \\ & \text{Iteration 57: } L_{57} = [2,0000000000000000001387778777656445675529539585113525390625; 2,01] \\ & \text{Iteration 58: } L_{58} = [2,00000000000000000006938893888282228377647697925567626953125; 2,01] \\ & \text{Iteration 59: } L_{59} = [2,000000000000000000034694469441411141888238489627838134765625; 2,01] \\ & \text{Iteration 60: } L_{60} = [2,0000000000000000000173472347207055709441192448139190673828125; 2,01] \\ & \text{Iteration 61: } L_{61} = [2,00000000000000000000867361736035278547205962240695953369140625; 2,01] \\ & \text{Iteration 62: } L_{62} = [2,000000000000000000004336808680176392736029811203479766845703125; 2,01] \\ & \text{Iteration 63: } L_{63} = [2,0000000000000000000021684043400881963680149056017398834228515625; 2,01] \\ & \text{Iteration 64: } L_{64} = [2,00000000000000000000108420217004409818400745280086994171142578125; 2,01] \\ & \text{Iteration 65: } L_{65} = [2,000000000000000000000542101085022049092003726400434970855712890625; 2,01] \\ & \text{Iteration 66: } L_{66} = [2,0000000000000000000002710505425110245460018632002174854278564453125; 2,01] \\ & \text{Iteration 67: } L_{67} = [2,00000000000000000000013552527125551227300093160010874271392822265625; 2,01] \\ & \text{Iteration 68: } L_{68} = [2,000000000000000000000067762635627756136500465800054371356964111328125; 2,01] \\ & \text{Iteration 69: } L_{69} = [2,0000000000000000000000338813178138780682502329000271856784820556640625; 2,01] \\ & \text{Iteration 70: } L_{70} = [2,00000000000000000000001694065890693903412511645001359283924102783203125; 2,01] \\ & \text{Iteration 71: } L_{71} = [2,000000000000000000000008470329453469517062558225006796419620513916015625; 2,01] \\ & \text{Iteration 72: } L_{72} = [2,0000000000000000000000042351647267347585312791125033982098102569580078125; 2,01] \\ & \text{Iteration 73: } L_{73} = [2,00000000000000000000000211758236336737926563955625169910400512847900390625; 2,01] \\ & \text{Iteration 74: } L_{74} = [2,000000000000000000000001058791181683689632819778125849552002564239501953125; 2,01] \\ & \text{Iteration 75: } L_{75} = [2,0000000000000000000000005293955908418448164098890629247760012821197509765625; 2,01] \\ & \text{Iteration 76: } L_{76} = [2,00000000000000000000000026469779542092240820494453146238800064105987548828125; 2,01] \\ & \text{Iteration 77: } L_{77} = [2,000000000000000000000000132348897710461204102472265731194000320529937744140625; 2,01] \\ & \text{Iteration 78: } L_{78} = [2,0000000000000000000000000661744488552306020512361328655970001602649688720703125; 2,01] \\ & \text{Iteration 79: } L_{79} = [2,00000000000000000000000003308722442761530102561806643279850008013248443603515625; 2,01] \\ & \text{Iteration 80: } L_{80} = [2,000000000000000000000000016543612213807650512809033216399250040066242218017578125; 2,01] \\ & \text{Iteration 81: } L_{81} = [2,0000000000000000000000000082718061069037752564045166081996250200331211090087890625; 2,01] \\ & \text{Iteration 82: } L_{82} = [2,00000000000000000000000000413590305345188762820225830409812501001656055450439453125; 2,01] \\ & \text{Iteration 83: } L_{83} = [2,000000000000000000000000002067951526725943814101129152049062505008280277252197265625; 2,01] \\ & \text{Iteration 84: } L_{84} = [2,0000000000000000000000000010339757633629719070505645760245312525041401386260986328125; 2,01] \\ & \text{Iteration 85: } L_{85} = [2,00000000000000000000000000051698788168148595352528228801226562625207006931304931640625; 2,01] \\ & \text{Iteration 86: } L_{86} = [2,000000000000000000000000000258493940840742976762641144006132813126035034656524658203125; 2,01] \\ & \text{Iteration 87: } L_{87} = [2,0000000000000000000000000001292469704203714883813205720030664065630175173282623291015625; 2,01] \\ & \text{Iteration 88: } L_{88} = [2,00000000000000000000000000006462348521018574419066028600153320328150875866413116455078125; 2,01] \\ & \text{Iteration 89: } L_{89} = [2,000000000000000000000000000032311742605092872095330143000766601640754379332065582275390625; 2,01] \\ & \text{Iteration 90: } L_{90} = [2,0000000000000000000000000000161558713025464360476650715003833008203771896640327911376953125; 2,01] \\ & \text{Iteration 91: } L_{91} = [2,00000000000000000000000000000807793565127321802383253575019165041018859483201639556884765625; 2,01] \\ & \text{Iteration 92: } L_{92} = [2,000000000000000000000000000004038967825636609011916267875095825205094297416008197784423828125; 2,01] \\ & \text{Iteration 93: } L_{93} = [2,0000000000000000000000000000020194839128183045059581339375479126025471487080040988922119140625; 2,01] \\ & \text{Iteration 94: } L_{94} = [2,00000000000000000000000000000100974195640915225297906696877395630127357435400204944610595703125; 2,01] \\ & \text{Iteration 95: } L_{95} = [2,000000000000000000000000000000504870978204576126489533484386978150636787177001024723052978515625; 2,01] \\ & \text{Iteration 96: } L_{96} = [2,0000000000000000000000000000002524354891022880632447667421934890753183935885005123615264892578125; 2,01] \\ & \text{Iteration 97: } L_{97} = [2,00000000000000000000000000000012621774455114403162238337109674453765919679425025618076324462890625; 2,01] \\ & \text{Iteration 98: } L_{98} = [2,000000000000000000000000000000063108872275572015811191685548372268829598397125128090381622314453125; 2,01] \\ & \text{Iteration 99: } L_{99} = [2,0000000000000000000000000000000315544361377860079055958427741861344147991985625640451908111572265625; 2,01] \\ & \text{Iteration 100: } L_{100} = [2,00000000000000000000000000000001577721806889300395279792138709306720739959928128202259540557861328125; 2,01] \\ & \text{Iteration 101: } L_{101} = [2,000000000000000000000000000000007888609034446501976398960693546533603699799640641011297702789306640625; 2,01] \\ & \text{Iteration 102: } L_{102} = [2,0000000000000000000000000000000039443045172232509881994803467732668018498998203205056488513946533203125; 2,01] \\ & \text{Iteration 103: } L_{103} = [2,00000000000000000000000000000000197215225861162549409974017338663340092494991016025282442569732666015625; 2,01] \\ & \text{Iteration 104: } L_{104} = [2,000000000000000000000000000000000986076129305581247049870086693316700462474955080126412127848663330078125; 2,01] \\ & \text{Iteration 105: } L_{105} = [2,0000000000000000000000000000000004930380646527906235249350433466583502312374775400632060639243316650390625; 2,01] \\ & \text{Iteration 106: } L_{106} = [2,00000000000000000000000000000000024651903232639531176246752167332917511561873877003160303196216583251953125; 2,01] \\ & \text{Iteration 107: } L_{107} = [2,000000000000000000000000000000000123259516163197655881233760836658350057809369385015801515981082916259765625; 2,01] \\ & \text{Iteration 108: } L_{108} = [2,0000000000000000000000000000000000616297580815988279406168804183291750289046846925079007579905414581298828125; 2,01] \\ & \text{Iteration 109: } L_{109} = [2,00000000000000000000000000000000003081487904079941397030844020916458751445234234625395037899527072906494140625; 2,01] \\ & \text{Iteration 110: } L_{110} = [2,000000000000000000000000000000000015407439520399706985154220104582293757226171173126975189497635364532470703125; 2,01] \\ & \text{Iteration 111: } L_{111} = [2,0000000000000000000000000000000000077037197601998534925771100522911468786130855865634875947488176822662353515625; 2,01] \\ & \text{Iteration 112: } L_{112} = [2,00000000000000000000000000000000000385185988009992674628555502614557343930654279328174379737440884113311767578125; 2,01] \\ & \text{Iteration 113: } L_{113} = [2,000000000000000000000000000000000001925929940049963373142777513072786719653271396640871898687204420566558837890625; 2,01] \\ & \text{Iteration 114: } L_{114} = [2,0000000000000000000000000000000000009629649700249816867138875653613933598263569833204359493436022102832794189453125; 2,01] \\ & \text{Iteration 115: } L_{115} = [2,00000000000000000000000000000000000048148248501249084335694378268069667991317849166021797467180110514163970947265625; 2,01] \\ & \text{Iteration 116: } L_{116} = [2,000000000000000000000000000000000000240741242506245421683218913040348339956589245830108987335900552570819854736328125; 2,01] \\ & \text{Iteration 117: } L_{117} = [2,0000000000000000000000000000000000001203706212531227108416094565201741699782946229150544936679502762854099273681640625; 2,01] \\ & \text{Iteration 118: } L_{118} = [2,00000000000000000000000000000000000006018531062656135542080472826008708498914731145752724683397513814270496368408203125; 2,01] \\ & \text{Iteration 119: } L_{119} = [2,000000000000000000000000000000000000030092655313280677710402364130043542494573655728763623416987569071352481842041015625; 2,01] \\ & \text{Iteration 120: } L_{120} = [2,0000000000000000000000000000000000000150463276566403388552011820650217712472868278643818117084937845356762409210205078125; 2,01] \\ & \text{Iteration 121: } L_{121} = [2,00000000000000000000000000000000000000752316382832016692760059103251088562364341393219090585424689226783812046051025390625; 2,01] \\ & \text{Iteration 122: } L_{122} = [2,000000000000000000000000000000000000003761581914160083346300295516255442811821706966095452927123446133919060230255126953125; 2,01] \\ & \text{Iteration 123: } L_{123} = [2,0000000000000000000000000000000000000018807909570800416731501477581277214059108534830477264635617230669595301151275634765625; 2,01] \\ & \text{Iteration 124: } L_{124} = [2,00000000000000000000000000000000000000094039547854002083657500737906386070295542674152386323178086153347976505756378173828125; 2,01] \\ & \text{Iteration 125: } L_{125} = [2,000000000000000000000000000000000000000470197739270010418287503689531930351477713370761931615890430766739882528781890869140625; 2,01] \\ & \text{Iteration 126: } L_{126} = [2,000000000000000000000000000000000000000235098869635005209143751844765965175738856685359655787945$$